Noncollinear drag force in Bose-Einstein condensates with Weyl spin-orbit coupling

Renyuan Liao,[1] Oleksandr Fialko,[2] Joachim Brand,[3] and Ulrich Zülicke[4]
[1]*Fujian Provincial Key Laboratory for Quantum Manipulation and New Energy Materials, College of Physics and Energy, Fujian Normal University, Fuzhou 350108, China*
[2]*Dodd-Walls Centre for Photonics and Quantum Technology, Institute of Natural and Mathematical Sciences and Centre for Theoretical Chemistry and Physics, Massey University, Auckland 0745, New Zealand*
[3]*Dodd-Walls Centre for Photonics and Quantum Technology, New Zealand Institute for Advanced Study, and Centre for Theoretical Chemistry and Physics, Massey University, Auckland 0745, New Zealand*
[4]*School of Chemical and Physical Sciences and Dodd-Walls Centre for Photonics and Quantum Technology, Victoria University of Wellington, PO Box 600, Wellington 6140, New Zealand*
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We consider the motion of a pointlike impurity through a three-dimensional two-component Bose-Einstein condensate subject to Weyl spin-orbit coupling. Using linear-response theory, we calculate the drag force felt by the impurity and the associated anisotropic critical velocity from the spectrum of elementary excitations. The drag force is shown to be generally not collinear with the velocity of the impurity. This unusual behavior is a consequence of condensation into a finite-momentum state due to the spin-orbit coupling.

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Degenerate quantum gases of neutral atoms [1,2], polaritons [3], as well as the recently discovered condensation of light [3] have provided new opportunities for studying superfluidity. One of the most remarkable manifestations of superfluidity is that impurities immersed into such systems propagate without dissipation if their velocities do not exceed the Landau critical velocity [4]

$$v_c = \min_q \left[\frac{\omega(q)}{q}\right], \tag{1}$$

where $\omega(q)$ is the spectrum of elementary excitations. As long as the impurity moves slower than the critical velocity, the superfluid cannot absorb any of its energy and therefore the impurity motion is frictionless. Experiments with atomic Bose-Einstein condensates (BECs) have provided evidence for a critical velocity associated with emission of elementary excitations [5–9] as well as more complex excitations like vortices and solitons [10–14]. Intense theoretical efforts [15–28] have been undertaken to study the stability of superfluidity and explain the mechanisms of dissipation in BECs.

Recently, the experimental realization of various synthetic gauge fields including one-dimensional and two-dimensional spin-orbit coupling (SOC) in quantum gases [29–32] enabled the prediction of a number of novel interesting properties in these new types of condensates [31,33–37]. Such superfluids differ from the conventional ones in that they are condensations of Bose atoms at some finite momentum. As SOC breaks the Galilean invariance of the system [33,37–39], the applicability of the Landau criterion for superfluidity in the new BECs becomes questionable. This calls for a better understanding of the critical velocity and dissipation mechanism of this new superfluid. Recent work has focused on the special case of a planar spin-orbit-coupled atom gas with spin-independent interactions where the system's high symmetry caused the superfluid critical velocity to vanish except when the impurity was moving in the direction opposite to the condensate wave vector [39]. Here we examine superfluidity in a *three-dimensional* two-component Bose gas with Weyl SOC [40] by studying the drag force felt by a moving pointlike impurity [41]. In further contrast to previous work [39], we also allow for the more general case where interactions break spin-rotational invariance. The drag force is calculated within linear-response theory from the elementary excitation spectrum and can be related to the dynamical structure factor [17]. It turns out to depend strongly on the impurity-velocity direction, thus demonstrating the anisotropy of the critical velocity. We also find that the drag force is not generally collinear with the velocity of the impurity, which is a feature of spin-orbit-coupled BECs [39] that is in stark contrast to a conventional superfluid. This fact can be used to probe SOC by the scattering of heavy molecules by the condensate.

The second-quantized Hamiltonian for the BEC with a pointlike impurity moving with velocity v is

$$H = \int d^3\mathbf{r}\, \Psi^\dagger\left[\left(-\frac{\hbar^2\nabla^2}{2m} - \mu\right)I + \lambda\boldsymbol{\sigma}\cdot\mathbf{P}\right]\Psi + \int d^3\mathbf{r}\left[\frac{g}{2}n^2 + (g_{\uparrow\downarrow} - g)n_\uparrow n_\downarrow + g_i n\delta(\mathbf{r} - \mathbf{v}t)\right]. \tag{2}$$

Here $\Psi(\mathbf{r}) = (\psi_\uparrow,\psi_\downarrow)^T$ is the two-component condensate quantum field, I is the 2×2 identity matrix, $n(\mathbf{r}) = n_\uparrow(\mathbf{r}) + n_\downarrow(\mathbf{r}) \equiv \psi_\uparrow^\dagger\psi_\uparrow + \psi_\downarrow^\dagger\psi_\downarrow$ is the density operator, μ is the chemical potential, λ is the strength of the spin-orbit coupling, g_i is the particle-impurity coupling constant, and the strengths of the intraspecies interaction and interspecies interaction are g and $g_{\uparrow\downarrow}$, respectively. For brevity, we set $\hbar = 2m = 1$ from now on.

The time-dependent mean-field Gross-Pitaevskii (GP) equation found from Eq. (2) reads

$$[(-i\partial_t - \nabla^2 - \mu)I - i\lambda\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}]\Psi_0 + gn_0\Psi_0 + (g_{\uparrow\downarrow} - g)\begin{pmatrix} n_{0\downarrow} & 0 \\ 0 & n_{0\uparrow}\end{pmatrix}\Psi_0 + g_i\delta(\mathbf{r} - \mathbf{v}t)\Psi_0 = 0. \tag{3}$$

In absence of the impurity, the ground state has been found to exhibit rich Skyrmion-type structures for trapped systems

[42–44], whereas it is of plane-wave type for homogeneous systems [45,46] that we focus on in this work. To proceed, we split the field $\Psi_0(\mathbf{r},t) = \Phi_0(\mathbf{r}) + \Phi(\mathbf{r},t)$, where $\Phi_0(\mathbf{r}) = \sqrt{\frac{n_0}{2}}(1,1)^T e^{i\mathbf{K}\cdot\mathbf{r}}$ is the mean-field solution without the impurity, and $\Phi(\mathbf{r},t)$ is the perturbation caused by the impurity. Without loss of generality, we choose the condensation momentum to be $\mathbf{K} = (-\lambda/2,0,0)$, and the chemical potential becomes $\mu = n_0(g + g_{\uparrow\downarrow})/2 - K^2$. Linearizing GP in Φ, we obtain

$$\left[\left(-i\partial_t - \nabla^2 + K^2 + \frac{gn_0}{2}\right)I - i\lambda\boldsymbol{\sigma}\cdot\nabla + \frac{g_{\uparrow\downarrow}n_0}{2}\sigma_x\right]\Phi + \left(\frac{g_{\uparrow\downarrow}n_0}{2}\sigma_x + \frac{gn_0}{2}I\right)e^{2i\mathbf{K}\cdot\mathbf{r}}\Phi^* + g_i\delta(\mathbf{r}-\mathbf{v}t)\Phi_0 = 0. \tag{4}$$

The ansatz $\Phi(\mathbf{r},t) = e^{i\mathbf{K}\cdot\mathbf{r}}\sum_{\mathbf{q}}\varphi_{\mathbf{q}}e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{v}t)}$ yields

$$[(-\mathbf{q}\cdot\mathbf{v} + q^2 + 2K^2 + 2\mathbf{K}\cdot\mathbf{q})I + \lambda\boldsymbol{\sigma}\cdot(\mathbf{K}+\mathbf{q})]\varphi_{\mathbf{q}} + \left(\frac{gn_0}{2}I + \frac{g_{\uparrow\downarrow}n_0}{2}\sigma_x\right)(\varphi_{\mathbf{q}} + \varphi^*_{-\mathbf{q}}) = -g_i\sqrt{\frac{n_0}{2}}\begin{pmatrix}1\\1\end{pmatrix}. \tag{5}$$

Combining Eq. (5) with its complex conjugate, we obtain

$$[\mathcal{H}_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{v}\mathcal{I}]\begin{pmatrix}\varphi_{\mathbf{q}\uparrow}\\ \varphi_{\mathbf{q}\downarrow}\\ \varphi^*_{-\mathbf{q}\uparrow}\\ \varphi^*_{-\mathbf{q}\downarrow}\end{pmatrix} = -g_i\sqrt{\frac{n_0}{2}}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \tag{6}$$

with the matrices

$$\mathcal{H}_{\mathbf{q}} = \begin{pmatrix}M_{\mathbf{q}} & B\\ B & M^*_{-\mathbf{q}}\end{pmatrix}, \quad \mathcal{I} = \begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}. \tag{7}$$

Here $B = \frac{gn_0}{2}I + \frac{g_{\uparrow\downarrow}n_0}{2}\sigma_x$ and

$$M_{\mathbf{q}} = (q^2 + 2K^2 + 2\mathbf{K}\cdot\mathbf{q})I + \lambda\boldsymbol{\sigma}\cdot(\mathbf{K}+\mathbf{q}) + B. \tag{8}$$

Solving Eq. (6) for $\varphi_{\mathbf{q}\uparrow}$ and $\varphi_{\mathbf{q}\downarrow}$, the force acting on the impurity is found as

$$\mathbf{F} = -\int d^3\mathbf{r}\,\Psi_0^\dagger\nabla[g_i\delta(\mathbf{r}-\mathbf{v}t)]\Psi_0 \equiv g_i\nabla|\Psi_0(\mathbf{r},t)|^2_{\mathbf{r}=\mathbf{v}t} \tag{9}$$

$$\approx g_i\sqrt{\frac{n_0}{2}}\sum_{\mathbf{q}} i\mathbf{q}(\varphi_{\mathbf{q}\uparrow} + \varphi_{\mathbf{q}\downarrow} + \varphi^*_{-\mathbf{q}\uparrow} + \varphi^*_{-\mathbf{q}\downarrow}),$$

$$= -\frac{g_i^2 n_0}{2}\sum_{\mathbf{q}} i\mathbf{q}\sum_{ij}([\mathcal{H}_{\mathbf{q}} - (\mathbf{q}\cdot\mathbf{v} + i0^+)\mathcal{I}]^{-1})_{ij}. \tag{10}$$

The infinitesimal imaginary part was added following the usual causality rule [17,20,39].

According to the fluctuation-dissipation theorem, the drag force should be related to the fluctuation properties (i.e., the spectrum of elementary excitations) of the unperturbed system. This fact will assist us to analyze the drag force in more detail. We consider the system without the impurity by setting $g_i = 0$ in Eq. (2). The partition function of the system can be conveniently cast as imaginary time field integral [47] $\mathbf{Z} = \int d[\psi^*_\sigma,\psi_\sigma]e^{-S[\psi^*_\sigma,\psi_\sigma]}$ with the action given by $S = \int_0^\beta d\tau[\int d\mathbf{r}\sum_\sigma \psi^*_\sigma\partial_\tau\psi_\sigma + H(\psi^*_\sigma,\psi_\sigma)]$, where $\tau = it$ is the imaginary time. We replace the Bose field with a static part and a fluctuating part as $\psi_\sigma = \phi_{\sigma 0} + \phi_\sigma$ [48,49]. Within the Bogoliubov approximation, we expand the action up to quadratic orders in the fluctuating fields, and approximate the action as $S \approx S_{\text{eff}} = S_0 + S_g$. Here S_0 is the saddle point action containing only the static fields $\phi_{\sigma 0}$, and S_g is the Gaussian action containing fluctuating fields ϕ_σ of quadratic orders. By defining column vectors $\Xi_q = (\phi_{\mathbf{K}+\mathbf{q}\uparrow},\phi_{\mathbf{K}+\mathbf{q}\downarrow},\phi^*_{\mathbf{K}-\mathbf{q}\uparrow},\phi^*_{\mathbf{K}-\mathbf{q}\downarrow})^T$, we may write the Gaussian action in a compact form as $S_g = \frac{1}{2}\sum_{\mathbf{q},iw_n}\Xi_q^\dagger\mathcal{G}^{-1}\Xi_q$, where $q = (\mathbf{q},iw_n)$ with $w_n = 2n\pi/\beta$ being bosonic Matsubara frequencies. The inverse Green's function is given by $\mathcal{G}^{-1}(\mathbf{q},iw_n) = \mathcal{H}_{\mathbf{q}} - i\omega_n\mathcal{I}$. Comparison with Eq. (10) yields the drag force in terms of the Green's function of the unperturbed system:

$$\mathbf{F} = -\frac{g_i^2 n_0}{2}\sum_{\mathbf{q}} i\mathbf{q}\sum_{ij}\mathcal{G}_{ij}(\mathbf{q},iw_n \to \mathbf{q}\cdot\mathbf{v} + i0^+). \tag{11}$$

Within Bogoliubov theory, the dynamical structure factor of the unperturbed system can be evaluated as [48]

$$S(\mathbf{q},iw_n) = N^{-1}\langle\delta\rho(\mathbf{q},iw_n)^\dagger\delta\rho(\mathbf{q},iw_n)\rangle_0 = \sum_{i,j}\mathcal{G}_{ij}(\mathbf{q},iw_n). \tag{12}$$

Thus the drag force can be also expressed in terms of the dynamic structure factor:

$$\mathbf{F} = -\frac{g_i^2 n_0}{2}\sum_{\mathbf{q}} i\mathbf{q}\,S(\mathbf{q},iw_n \to \mathbf{q}\cdot\mathbf{v} + i0^+). \tag{13}$$

The spectrum $\omega_i(\mathbf{q})$ of elementary excitations is found by solving $\text{Det}[\mathcal{G}^{-1}(\mathbf{q},iw_n)] = 0$ with subsequent analytic continuation $i\omega_n \to \omega_i(\mathbf{q})$. Using this fact, we arrive at the following expression for the drag force:

$$\mathbf{F} = 2\pi g_i^2 n_0\sum_{\mathbf{q}}\mathbf{q}\sum_{i=1}^{4}\frac{J(\mathbf{q},\omega_i(\mathbf{q}))\delta(\mathbf{q}\cdot\mathbf{v} - \omega_i(\mathbf{q}))}{\prod_{j\neq i}[\omega_i(\mathbf{q}) - \omega_j(\mathbf{q})]}. \tag{14}$$

Here we used the abbreviation

$$J(\mathbf{q},iw_n) = q^2(iw_n + 2\lambda q_x)^2 - (q^2+\lambda^2)(q^4 + \lambda^2 q_x^2) - (g - g_{\uparrow\downarrow})n_0(q^4 + \lambda^2 q^2 - \lambda^2 q_y^2). \tag{15}$$

Let us first calculate the drag force in the absence of SOC by setting $\lambda = 0$. In this case, the four branches of excitations are the Bogoliubov-type modes $\omega^0_{1,2} = \pm q\sqrt{q^2 + (g + g_{\uparrow\downarrow})n_0}$ and $\omega^0_{3,4} = \pm q\sqrt{q^2 + (g - g_{\uparrow\downarrow})n_0}$. The former is the spectrum of density waves propagating with the speed of sound $c = \sqrt{(g + g_{\uparrow\downarrow})n_0}$, while the latter is the spectrum of spin waves. Using these analytical expressions, we evaluate the sum in Eq. (14) and find the drag force $\mathbf{F}_0 = \mathbf{v}g_i^2 n_0 v(1 - c^2/v^2)^2/(16\pi)\Theta(v - c)$, in agreement with Ref. [17]. The drag force in this case is collinear with the impurity's direction of motion. Note also that the lower-lying spin-wave mode is not excited because the impurity couples only to density waves.

The presence of SOC modifies the above result. Due to condensation into a finite-momentum state, the ground state breaks rotational symmetry, and the additionally broken spin-rotational invariance arising from $g_{\uparrow\downarrow} \neq g$ guarantees that the condensate state is stable against small fluctuations, including the motion of an impurity. The broken rotational symmetry also renders the spectrum of elementary excitations anisotropic. For our choice of condensate momentum, the spectrum is invariant under flipping the direction of q_y and/or q_z, namely $\omega_i(q_x,\pm q_y,\pm q_z) = \omega_i(q_x,q_y,q_z)$. As a result, the x direction is distinguished from the y and z axes. To be specific, let's assume that the impurity moves along the z axis. We can write $\delta(\mathbf{q}\cdot\mathbf{v} - \omega_i) = \delta(q - q_{z0})/|v_z - \partial\omega_i/\partial q_z|$, where $q_{z0} = h_i(q_x,q_y^2)$ is some function reflecting symmetry properties, and the detailed form of h_i is unimportant for our further analysis. Carrying out the integration in Eq. (14), one immediately finds that F_y vanishes and both F_x and F_z survive, by considering the integration of odd or even functions within a symmetrical interval. This argument can be repeated for different directions of the velocity yielding an additional contribution to the drag force along the x axis. Therefore, in addition to the conventional force component along the velocity vector, a force component along x axis is generated, due to the asymmetrical excitation spectrum with regard to x axis. For small spin-orbit coupling, corrections in the drag force brought about by SOC may be estimated by performing an expansion in the parameter λ. Up to the first nonvanishing order in the SOC strength, after lengthy but straightforward calculations, we obtain $\mathbf{F} = \mathbf{F}_0 + \mathbf{F}_\parallel + \mathbf{F}_x$ with $\mathbf{F}_\parallel \approx \hat{v}O(\lambda^2)$ and $\mathbf{F}_x\hat{x}O(\lambda^3)$. Evidently, SOC produces an additional drag along the direction of condensation.

To substantiate the above argument we now calculate the drag force (14) numerically. For our numerics we choose gn_0 as the energy scale and $\sqrt{gn_0}$ as the momentum scale. Let us first consider simple situations where the velocity of the moving impurity is along the x, y, and z axes, respectively. Results obtained for these cases are shown in Fig. 1. When the velocity is along the x axis, the drag force is also along x axis, and is asymmetrical between positive and negative direction of velocity, reflecting the spontaneously broken symmetry of the ground state with finite condensate momentum in the negative x direction. It requires more force to drag the impurity against the direction of condensation than along with it. When the velocity is along the y axis or the z axis, the magnitude of force does not change upon reversing the direction of the velocity. However, it is remarkable that in both cases a nonvanishing force component along the x axis emerges. Figure 1 also shows that there exists a critical velocity v_c below which there is no drag force. Its magnitude decreases when the strength of SOC is increased. We calculated the critical velocity in three orthogonal planes by examining the lower bound of the velocity where the drag force becomes nonzero. These results are presented in Fig. 2. As shown in panel (a), the critical velocity along the *negative* x axis remains unchanged as SOC strength is increased, however, it decreases significantly along the *positive* x axis. As can be seen in panel (b), the critical velocity in the yz plane is slightly deformed from a circle, signaling the inequivalence between y and z directions.

Let us now examine the drag force in more detail. Fixing the velocity of the impurity to lie in the xy plane we show the behavior of the corresponding drag force in Fig. 3. Here we define the azimuth of the drag force to be $\phi_F = \arg(F_x + iF_y)$,

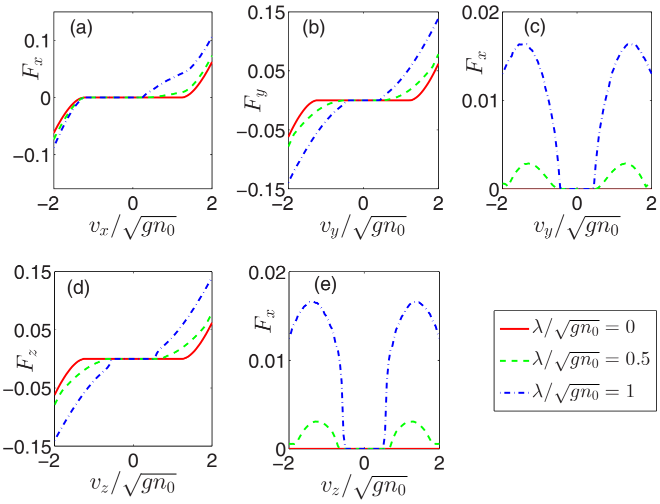

FIG. 1. Drag force (measured in units of $g_i^2 n_0$) experienced by an impurity moving in a Weyl-spin-orbit-coupled BEC with condensate wave vector parallel to the negative x direction and $g_{\uparrow\downarrow}/g = 1/2$. Results for different spin-orbit coupling strength λ are given. For the case of the impurity velocity being parallel to the x axis, only the x component F_x of the drag force is finite even for $\lambda \neq 0$ [panel (a)]. When the velocity is along the y axis, the drag force has components along the y axis [panel (b)] and, for finite λ, also along the x axis [panel (c)]. When the velocity is along the z axis, the drag force has finite components in the z and x directions when $\lambda \neq 0$ [panels (d) and (e)].

i.e., the angle in the xy plane, and the azimuth of the velocity to be $\phi_v = \arg(v_x + iv_y)$. In panel (a), the x component for the scaled drag force F_x has the symmetry of $F_x(\pi - \phi_v) = F_x(\pi + \phi_v)$; namely it has reflection symmetry with respect to the x axis. The y component F_y entails the symmetry of $F_y(\pi - \phi_v) = -F_y(\pi + \phi_v)$, as indicated in panel (b). The z component of the drag force vanishes. In panel (c), we show the difference between the azimuth of the drag force and the

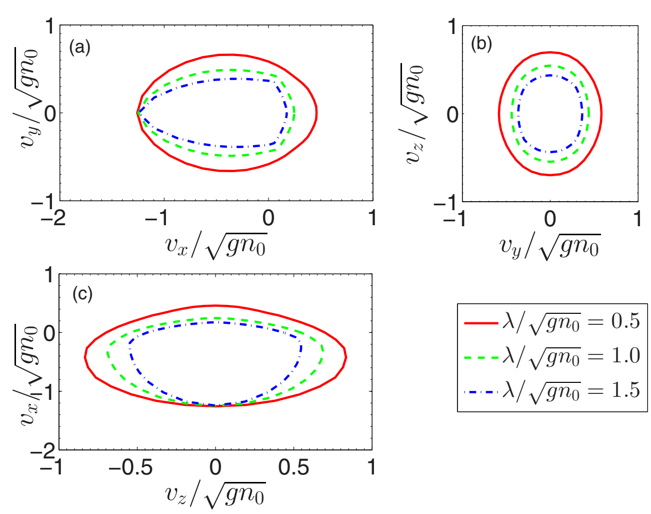

FIG. 2. Directional dependence of the critical velocity for an impurity moving in a Weyl-spin-orbit-coupled BEC with condensate wave vector parallel to the negative x direction and $g_{\uparrow\downarrow}/g = 1/2$.

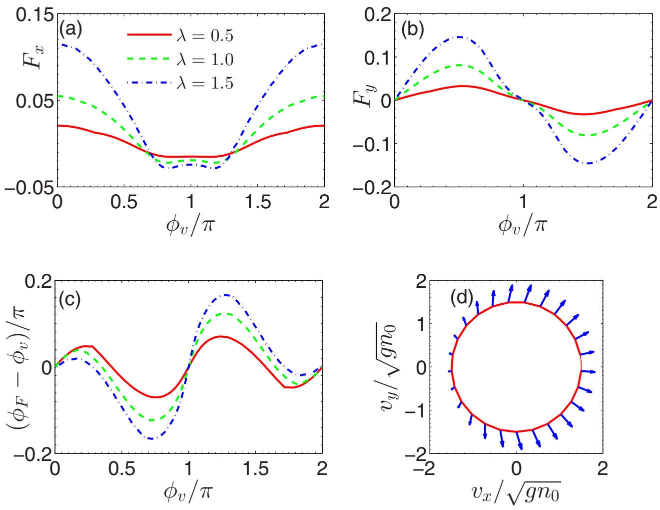

FIG. 3. Cartesian components of the drag force for an impurity moving in the xy plane at azimuthal angle ϕ_v with speed $v = 1.5\sqrt{gn_0}$: (a) F_x; (b) F_y. (c) Difference between the azimuthal angles of drag force and velocity; $\phi_F - \phi_v$. (d) Visualization of the drag force for $\lambda = 1.5\sqrt{gn_0}$. Blue arrows indicate the force vectors for velocities corresponding to points on the red circle. Here $g_{\uparrow\downarrow}/g = 1/2$ was assumed.

azimuth of the velocity. It is quite remarkable that the direction of the drag force is not aligned with the velocity, as is the case in a conventional superfluid. For a better visualization, we show the force vector in panel (d), where the arrow sitting on constant circle of speed indicates the force vector.

Now we fix the velocity vector to lie in the yz plane. The drag force is shown in Fig. 4. Panel (a) illustrates that the drag force has an x component that is independent of the direction of the velocity within the yz plane for a fixed small spin-orbit coupling strength λ. For large λ, F_x oscillates slightly with varying directions of the velocity in yz

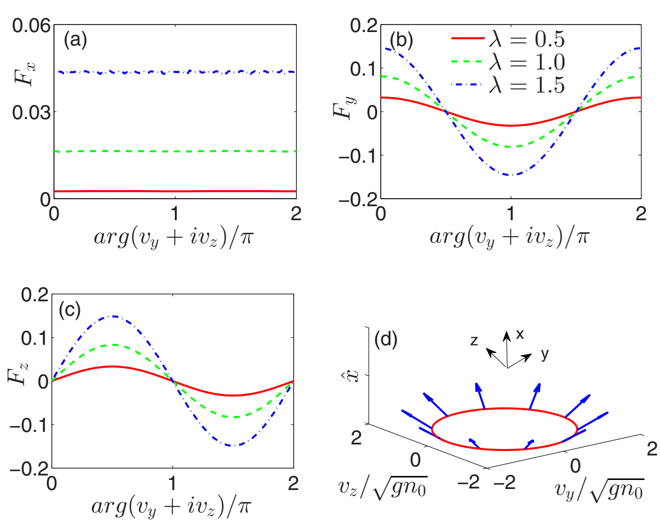

FIG. 4. Cartesian components of the drag force for an impurity moving in the yz plane with speed $v = 1.5\sqrt{gn_0}$: (a) F_x; (b) F_y; (c) F_z. (d) Visualization of the force vector for $\lambda = 1.5\sqrt{gn_0}$. The force vector for different velocity directions is indicated by arrows sitting on the circle of constant speed. Here $g_{\uparrow\downarrow}/g = 1/2$ was assumed.

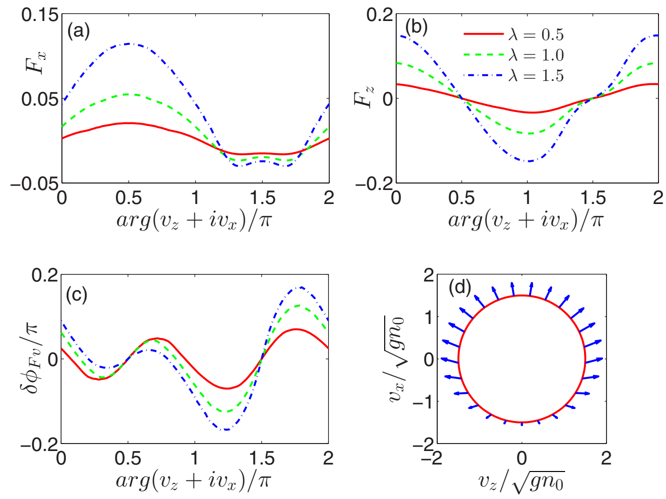

FIG. 5. Cartesian components of the drag force for an impurity moving in the zx plane with speed $v = 1.5\sqrt{gn_0}$: (a) F_x; (b) F_z. (c) Difference between the azimuthal angles of the drag force and the velocity; $\delta\phi_{Fv} \equiv \arg(F_z + i F_x) - \arg(v_z + i v_x)$. (d) Visualization of the force vector for $\lambda = 1.5\sqrt{gn_0}$, with force vectors indicated by arrows sitting on the circle of constant speed in the zx plane. Here $g_{\uparrow\downarrow}/g = 1/2$ was assumed.

plane. This inequivalence between y axis and z axis is due to the breaking of spin-rotational invariance by the nonlinear interaction potential $g - g_{\uparrow\downarrow} \neq 0$. In panel (b), F_y is shown to be symmetric with respect to y axis while antisymmetric with respect to z axis. In panel (c), F_z is antisymmetric with respect to y axis and symmetric with respect to the z axis. Interestingly, as can been seen in panel (d), the direction of the force is not lying in the yz plane, but is tilted towards the x axis.

Results for the case when the velocity vector lies in the zx plane are shown in Fig. 5. The behavior of the drag force looks quite similar to the situation when the velocity is in the xy plane. Panel (a) illustrates that F_x is symmetric with respect to x axis, while panel (b) shows that F_z is antisymmetric with respect to the z axis. As seen in panel (c), the difference of azimuthal angles for the force and velocity is antisymmetric with respect to the z axis. In panel (d), the force vector is visualized.

In summary, we have studied the motion of a pointlike impurity in a three-dimensional two-component BEC with Weyl SOC. We calculated the drag force and the associated critical velocity. At small SOC strength, we showed that the drag force can be decomposed into two parts. One is along the direction of the moving velocity, and the other one is along the direction of the condensation momentum. Hence, unlike in non-spin-orbit-coupled superfluids, the drag force is not generally collinear with the velocity of the impurity [39].

The unusual features revealed in our work can be utilized to probe SOC in bosonic superfluids. For example, having a finite critical velocity for any direction of impurity motion is indicative of spin-rotationally noninvariant interactions. Furthermore, finding the direction for which the drag force is collinear with the impurity's velocity will allow one to determine experimentally the direction of the condensate wave vector. The asymmetry of the force magnitude with respect to velocity reversal parallel to this direction can serve as an independent check. Finally, the existence of a drag-force component that is transverse to the direction the impurity moves in should lend itself to simulating Hall-effect-related phenomena [50] and could enable the design of novel atomtronic circuitry [51].

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