Reservoir-engineered entanglement in a hybrid modulated three-mode optomechanical system

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(Received 26 December 2017; published 10 April 2018)

We propose an effective approach for generating highly pure and strong cavity-mechanical entanglement (or optical-microwave entanglement) in a hybrid modulated three-mode optomechanical system. By applying two-tone driving to the cavity and modulating the coupling strength between two mechanical oscillators (or between a mechanical oscillator and a transmission line resonator), we obtain an effective Hamiltonian where an intermediate mechanical mode acting as an engineered reservoir cools the Bogoliubov modes of two target system modes via beam-splitter-like interactions. In this way, the two target modes are driven to two-mode squeezed states in the stationary limit. In particular, we discuss the effects of cavity-driving detuning on the entanglement and the purity. It is found that the cavity-driving detuning plays a critical role in the goal of acquiring highly pure and strongly entangled steady states.

DOI: 10.1103/PhysRevA.97.042314

I. INTRODUCTION

Theoretical explorations of a quantum optomechanical system began in the 1990s, including several aspects such as the squeezing of light [1,2], quantum nondemolition detection of the light intensity [3,4], preparation of nonclassical states [5–7], and so on. Ever since the optical feedback cooling scheme based on the radiation-pressure force was first experimentally demonstrated in 1999 [8], cavity optomechanics has attracted much interest, and fruitful progress has been made. Apart from its potential applications in building highly sensitive sensors and in testing macroscopic quantum mechanics [9], cavity optomechanics can also serve as a light-matter interface to convert information among different systems such as atoms or atomic ensembles [10,11], Bose-Einstein condensates [12,13], superconducting solid state qubits [14], etc.

To date, a variety of experimental optomechanical setups have been reported, e.g., whispering gallery microdisks [15,16] and microspheres [17,18], membranes [19] or nanorods [20] inside Fabry-Pérot cavities, a nanomechanical beam inside a superconducting transmission line microwave cavity [21], etc. Notably, the hybrid optomechanical system consisting of different physical components possesses the distinct advantages of each component, which may be beneficial for quantum-information processing (QIP). As experimentally demonstrated by Lee [22] and Winger [23], one can manipulate a mechanical nanoresonator via both the opto- and electromechanical interactions, which may provide a platform to entangle microwave and optical fields [24].

In this paper, we propose an effective approach for generating strong steady-state optomechanical entanglement (or optical-microwave entanglement), which is of great importance for both fundamental physics and applications in QIP. For a simple optomechanical system consisting of a laser-driven optical cavity and a vibrating end mirror, the entanglement between the cavity field and the mechanical resonator can be induced by the radiation pressure. However, the amount of created entanglement is largely limited due to environmental noises and the stability constraints of systems [25]. To enhance the entanglement strength, a feasible way is to apply a suitable time modulation to the driving laser [26,27]. The method is also effective in three-mode [28–30] or four-mode [31,32] optomechanical systems. Another promising approach for creating strong entanglement or squeezing is to induce an effective engineered reservoir by pumping the optomechanical systems with proper blue and red detuned lasers [31–40], which is highly attractive from an experimental point of view. As far as we know, previous studies mostly focused on enhancing entanglement between two cavity fields [33,34] or two mechanical oscillators [30–32,35–38]. Here, inspired by the approach in Ref. [36], which has been experimentally demonstrated recently [41], we propose to use both time modulation and reservoir engineering techniques to generate highly pure optomechanical or optical-microwave entanglement that goes far beyond the entanglement limit based on coherent parametric coupling (i.e., ln2) [26,42,43]. In our hybrid three-mode optomechanical system, the intermediate mechanical mode acting as a cooling reservoir and the sum mode of the Bogoliubov modes of the other two system modes are coupled via the beam-splitter-like interaction. The sum mode in turn is coupled to the difference mode of the

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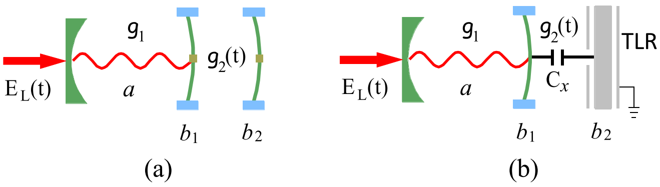

FIG. 1. Schematic representation of the system. An optical cavity mode a driven by a two-tone laser $E_L(t)$ is coupled to an intermediate mechanical mode b_1 with single-photon optomechanical coupling strength g_1. b_1 is in turn coupled, with a time-dependent coupling strength $g_2(t)$, to (a) another mechanical oscillator, or alternatively (b) a transmission line resonator b_2.

Bogoliubov modes. The swap interactions allow both the sum and the difference modes to be cooled via the dissipative dynamics of the intermediate mechanical mode, which is quite different from Refs. [33,34]. In Refs. [33,34], only one of the two Bogoliubov modes of the target modes is cooled while the other Bogoliubov mode is a dark mode that is not coupled to the engineered bath and thus cannot be cooled. Accordingly, the obtained steady states are two-mode squeezed thermal states. i.e., mixed states. On the contrary, our proposal allows the engineered bath to cool both Bogoliubov modes simultaneously. In this way, we are able to obtain a highly pure and strongly entangled steady state that is vital in the standard continuous-variable teleportation protocol [44,45]. Moreover, unlike the proposal in Ref. [36], which mainly focuses on the generation of steady-state mechanical-mechanical entanglement in the adiabatic limit, we show that steady optomechanical entanglement (or optical-microwave entanglement) can be maximized by choosing the proper ratio of the effective optomechanical couplings. We also discuss the critical role of the effective Bogoliubov-mode coupling (i.e., the frequency detuning between the cavity and the pumping) on the steady-state entanglement and purity, which is not considered in Ref. [36].

II. THE MODEL

As shown in Fig. 1, a hybrid modulated three-mode optomechanical system is composed of an optical cavity mode a and two mechanical oscillators b_1 and b_2 [see Fig. 1(a)]; or a cavity mode a, a mechanical oscillator b_1, and a transmission line resonator b_2 [see Fig. 1(b)]. g_1 is the single-photon optomechanical coupling strength between the cavity mode a with frequency w_c and the intermediate mechanical mode b_1 with frequency w_1. The cavity is driven by a two-tone laser $E_L(t)$. $g_2(t)$ is the time-dependent coupling between the intermediate mechanical mode b_1 and the second mechanical resonator (or the transmission line resonator) b_2 with frequency w_2. Here, the controllable mechanical-mechanical coupling $g_2(t)$ in Fig. 1(a) can be realized by using piezoelectrically induced parametric mode mixing [46] or by modulating the Coulomb interactions between the mechanical oscillators [40,47–50], while the mechanical-microwave coupling $g_2(t)$ in Fig. 1(b) may be achieved via the mechanical displacement-dependent capacitance C_x of the microwave cavity.

The system Hamiltonian reads (set $\hbar = 1$)

$$\begin{aligned} H = {}& w_c a^\dagger a + w_1 b_1^\dagger b_1 + w_2 b_2^\dagger b_2 + g_1(b_1 + b_1^\dagger) a^\dagger a \\ & + g_2(t)(b_1 + b_1^\dagger)(b_2 + b_2^\dagger) + H_{\rm dr}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} g_2(t) = {}& 2\big[g_2^A \cos(w_1 + w_2 + w_c - w_d)t \\ & + g_2^B \cos(w_1 - w_2 - w_c + w_d)t\big], \end{aligned} \tag{2}$$

and $H_{\rm dr}$ is the Hamiltonian of the two-tone driving with frequencies $w_d \pm w_1$,

$$H_{\rm dr} = (\epsilon_+^* e^{i w_1 t} + \epsilon_-^* e^{-i w_1 t}) e^{i w_d t} a + \text{H.c.} \tag{3}$$

Moving into a rotating frame by performing the unitary transformation $U = \exp\{-i[w_d a^\dagger a + w_1 b_1^\dagger b_1 + (w_2 + w_c - w_d) b_2^\dagger b_2]t\}$, we obtain

$$\begin{aligned} H_R = {}& U^\dagger H U - i U^\dagger \partial U/\partial t \\ = {}& \delta(a^\dagger a - b_2^\dagger b_2) + g_1(b_1 e^{-i w_1 t} + b_1^\dagger e^{i w_1 t}) a^\dagger a \\ & + g_2(t)(b_1 e^{-i w_1 t} + b_1^\dagger e^{i w_1 t})[b_2 e^{-i(w_2+\delta)t} \\ & + b_2^\dagger e^{i(w_2+\delta)t}] + [(\epsilon_+^* e^{i w_1 t} + \epsilon_-^* e^{-i w_1 t}) a + \text{H.c.}], \end{aligned} \tag{4}$$

where $\delta = w_c - w_d$ is the cavity-driving frequency detuning. Applying the displacement transformation $a = \bar{a}_+ e^{-i w_1 t} + \bar{a}_- e^{i w_1 t} + d$ to Eq. (4) in the strong driving case, we obtain the linearized Hamiltonian by discarding all nonlinear terms of the quantum fluctuations provided that the single-photon optomechanical coupling g_1 is small,

$$H_{\rm lin} = H_0 + H_1 + H_2, \tag{5}$$

with

$$H_0 = \delta(d^\dagger d - b_2^\dagger b_2), \tag{6a}$$

$$\begin{aligned} H_1 = {}& g_1[(\bar{a}_+ b_1 d + \bar{a}_- b_1 d^\dagger) + (\bar{a}_+ b_1 d^\dagger \\ & + \bar{a}_- b_1 d) e^{-2 i w_1 t}] + \text{H.c.}, \end{aligned} \tag{6b}$$

$$\begin{aligned} H_2 = {}& g_2^A \{b_1 b_2 [1 + e^{-2i(w_1 + w_2 + \delta)t}] \\ & + b_1 b_2^\dagger [e^{2i(w_2+\delta)t} + e^{-2 i w_1 t}]\} \\ & + g_2^B \{b_1 b_2 [e^{-2i(w_2+\delta)t} + e^{-2 i w_1 t}] \\ & + b_1 b_2^\dagger [1 + e^{-2i(w_1 - w_2 - \delta)t}]\} + \text{H.c.}, \end{aligned} \tag{6c}$$

where the classical cavity field amplitudes $\bar{a}_\pm$ are assumed to be real,

$$\bar{a}_\pm = i\epsilon_\pm/(-\kappa/2 - i\delta \pm i w_1), \tag{7}$$

and κ is the cavity decay rate. If we set $g_1 \bar{a}_+ = g_2^A = G_+$, $g_1 \bar{a}_- = g_2^B = G_-$, under the conditions $w_1, w_2, |w_1 - w_2 - \delta| \gg G_\pm$, all the nonresonant terms in the linearized Hamiltonian $H_{\rm lin}$ can be effectively neglected under the rotating-wave approximation,

$$H_{\rm RWA} = \delta(\beta_1^\dagger \beta_1 - \beta_2^\dagger \beta_2) + [G(\beta_1^\dagger + \beta_2^\dagger) b_1 + \text{H.c.}], \tag{8}$$

where the Bogoliubov modes β_1 and β_2 are unitary transformations of d and b_2, respectively,

$$\beta_1 = s(r) d s^\dagger(r) = d\cosh r + b_2^\dagger \sinh r, \tag{9a}$$

$$\beta_2 = s(r) b_2 s^\dagger(r) = b_2\cosh r + d^\dagger \sinh r. \tag{9b}$$

Here, $G=\sqrt{G_-^2-G_+^2}$ (we have assumed $G_+<G_-$ to ensure stability) and $s(r)=\exp[r(db_2-d^\dagger b_2^\dagger)]$ is the two-mode squeezing operator with the squeezing parameter $r=\tanh^{-1}(G_+/G_-)$. It is clear from Eq. (9) that the joint ground state of β_1 and β_2 is the two-mode squeezed vacuum state of the cavity mode d and the mechanical mode b_2. Introducing the sum mode and the difference mode of Bogoliubov modes

$$\beta_{\rm sum}=(\beta_1+\beta_2)/\sqrt{2},\ \beta_{\rm diff}=(\beta_1-\beta_2)/\sqrt{2}, \tag{10}$$

then the Hamiltonian in Eq. (8) becomes

$$H_{\rm RWA}=\delta\beta_{\rm sum}^\dagger\beta_{\rm diff}+\sqrt{2}G\beta_{\rm sum}^\dagger b_1+\text{H.c.}, \tag{11}$$

which is similar to that of Ref. [36]. Obviously, the sum mode $\beta_{\rm sum}$ is coupled to both the intermediate mechanical mode b_1 and the difference mode $\beta_{\rm diff}$ each via a beam-splitter-like interaction. Through the intermediate mechanical mode b_1 acting as an engineered reservoir, both the sum and difference modes, i.e., the two Bogoliubov modes β_1 and β_2, can be cooled to near ground state, generating two-mode squeezing between the cavity mode d and the mechanical mode b_2.

III. ENTANGLEMENT AND PURITY

The quantum Langevin equations governing the dynamics of the linearized system can be written as

$$\dot d = i[H_{\rm lin},d]-\frac{\kappa}{2}d+\sqrt{\kappa}d_{\rm in}, \tag{12a}$$

$$\dot b_j = i[H_{\rm lin},b_j]-\frac{\gamma_j}{2}b_j+\sqrt{\gamma_j}b_{j,\rm in}, \tag{12b}$$

where γ_j ($j=1,2$) is the damping rate for the jth mechanical oscillator, and $d_{\rm in}$ and $b_{j,\rm in}$ are independent zero mean vacuum input noise operators obeying the following correlation functions:

$$\langle d_{\rm in}(t)d_{\rm in}^\dagger(t')\rangle=(\bar n_d+1)\delta(t-t'), \tag{13a}$$

$$\langle d_{\rm in}^\dagger(t)d_{\rm in}(t')\rangle=\bar n_d\delta(t-t'), \tag{13b}$$

$$\langle b_{j,\rm in}(t)b_{j,\rm in}^\dagger(t')\rangle=(\bar n_j+1)\delta(t-t'), \tag{13c}$$

$$\langle b_{j,\rm in}^\dagger(t)b_{j,\rm in}(t')\rangle=\bar n_j\delta(t-t') \tag{13d}$$

with $\bar n_d$ and $\bar n_j$ being equilibrium mean thermal occupancies of the cavity and the jth mechanical baths, respectively.

Introducing the position and momentum quadratures for the bosonic modes and their input noises

$$Q_o=(o+o^\dagger)/\sqrt{2},\quad P_o=(o-o^\dagger)/(i\sqrt{2}), \tag{14}$$

with $o\in\{d,b_1,b_2,d_{\rm in},b_{1,\rm in},b_{2,\rm in}\}$ and the vectors of all quadratures

$$R=[Q_d,P_d,Q_{b_1},P_{b_1},Q_{b_2},P_{b_2}]^T, \tag{15a}$$

$$N=[\sqrt{\kappa}Q_{d_{\rm in}},\sqrt{\kappa}P_{d_{\rm in}},\sqrt{\gamma_1}Q_{b_{1,\rm in}}, \times\sqrt{\gamma_1}P_{b_{1,\rm in}},\sqrt{\gamma_2}Q_{b_{2,\rm in}},\sqrt{\gamma_2}P_{b_{2,\rm in}}]^T, \tag{15b}$$

the linearized quantum Langevin equations (12) can be written in a compact form,

$$\dot R = M(t)R+N. \tag{16}$$

Here, $M(t)$ is a 6×6 time-dependent matrix

$$M(t)=\begin{pmatrix} -\kappa/2 & \delta & {\rm Im}(G_1+G_2) & {\rm Re}(G_2-G_1) & 0 & 0\\ -\delta & -\kappa/2 & -{\rm Re}(G_2+G_1) & {\rm Im}(G_2-G_1) & 0 & 0\\ {\rm Im}(G_1-G_2) & {\rm Re}(G_2-G_1) & -\gamma_1/2 & 0 & {\rm Im}(G_3+G_4) & {\rm Re}(G_4-G_3)\\ -{\rm Re}(G_2+G_1) & -{\rm Im}(G_1+G_2) & 0 & -\gamma_1/2 & -{\rm Re}(G_3+G_4) & {\rm Im}(G_4-G_3)\\ 0 & 0 & {\rm Im}(G_3-G_4) & {\rm Re}(G_4-G_3) & -\gamma_2/2 & -\delta\\ 0 & 0 & -{\rm Re}(G_3+G_4) & -{\rm Im}(G_3+G_4) & \delta & -\gamma_2/2 \end{pmatrix}, \tag{17}$$

where Re and Im, respectively, denote the real and imaginary parts. G_1–G_4 are given by

$$G_1=G_++G_-e^{2iw_1t}, \tag{18a}$$

$$G_2=G_-+G_+e^{-2iw_1t}, \tag{18b}$$

$$G_3=G_+[1+e^{2i(w_1+w_2+\delta)t}]+G_-[e^{2i(w_2+\delta)t}+e^{2iw_1t}], \tag{18c}$$

$$G_4=G_-[1+e^{2i(w_1-w_2-\delta)t}]+G_+[e^{-2i(w_2+\delta)t}+e^{2iw_1t}]. \tag{18d}$$

Since the system is linearized, it remains Gaussian starting from an initial Gaussian state whose information-related properties can be fully described by the covariance matrix [51–53]. For our three-mode bosonic system, the covariance matrix σ is a 6×6 matrix with components defined as

$$\sigma_{j,k}=\langle R_jR_k+R_kR_j\rangle/2, \tag{19}$$

where R_k is the kth component of the vector of quadratures R in Eq. (15). From Eqs. (13), (15), and (16), we can derive a linear differential equation of the covariance matrix that is equivalent to the quantum Langevin equation (16) when only Gaussian states are relevant [26],

$$\dot\sigma = M(t)\sigma+\sigma M(t)^T+D. \tag{20}$$

Here, D is a diffusion matrix whose components are associated with the noise correlation functions [see Eq. (13)]

$$D_{j,k}\delta(t-t')=\langle N_j(t)N_k(t')+N_k(t')N_j(t)\rangle/2. \tag{21}$$

D is found to be diagonal,

$$D = \text{diag}\{\kappa(2\bar{n}_d + 1)/2, \kappa(2\bar{n}_d + 1)/2, \gamma_1(2\bar{n}_1 + 1)/2, \gamma_1(2\bar{n}_1 + 1)/2, \gamma_2(2\bar{n}_2 + 1)/2, \gamma_2(2\bar{n}_2 + 1)/2\}. \quad (22)$$

The general stability conditions of the linear differential equation [Eq. (16) or equally Eq. (20)] are determined by the corresponding homogeneous equation $\dot{R} = M(t)R$, which is fully characterized by the time-periodic coefficient matrix $M(t)$. Suppose that the period of the coefficient matrix $M(t)$ is $T > 0$, i.e., $M(t) = M(t + T)$. Let $\Pi(t)$ be a principal matrix solution of the homogeneous equation. The eigenvalues λ_j ($j = 1,2,\ldots,6$) of $\Lambda = \Pi^{-1}(0)\Pi(T)$ are called the characteristic multipliers or Floquet multipliers [54], where $\Pi(T)$ can be obtained by numerical integration with the initial condition $\Pi(0)$. The solutions of Eqs. (16) and (20) are stable if all Floquet multipliers satisfy $|\lambda_j| < 1$. For the special case of a time-independent coefficient matrix $M = M(t = 0)$ under the rotating-wave approximation, i.e., omitting all nonresonant terms in Eq. (5) [all time-dependent terms in Eq. (17)], the stability requirements can be readily inferred from the eigenvalues of the time-independent coefficient matrix M, i.e., all eigenvalues of M having negative real parts. The stability conditions will be carefully checked in all simulations throughout this paper.

For two-mode Gaussian states of the cavity mode d and the mechanical resonator b_2 of interest here, it is convenient to use the logarithmic negativity E_N as a measurement of the entanglement [55,56]. E_N can be computed from the reduced 4×4 covariance matrix σ_r for d and b_2 whose components are just the terms associated with d and b_2 only in the full covariance matrix σ. If we write σ_r in the form

$$\sigma_r = \begin{pmatrix} V_1 & V_c \\ V_c^T & V_2 \end{pmatrix}, \quad (23)$$

where V_1, V_2, and V_c are 2×2 subblock matrices of σ_r, the logarithmic negativity E_N is then given by

$$E_N = \max[0, -\ln(2\eta)], \quad (24)$$

with

$$\eta = 2^{-1/2}\{\Sigma - [\Sigma^2 - 4 \det \sigma_r]^{1/2}\}^{1/2}, \quad (25a)$$

$$\Sigma = \det V_1 + \det V_2 - 2 \det V_c. \quad (25b)$$

The purity of a two-mode Gaussian state described by a covariance matrix σ_r is simply given by

$$\mu = 1/(4\sqrt{\det \sigma_r}). \quad (26)$$

We next study the steady-state entanglement [$\dot{\sigma}(t) = 0$ in the stationary limit $t \gg 1/\kappa, \gamma_{1,2}$ if the system is stable] with the time-independent Hamiltonian in Eqs. (8) and (11) under the rotating-wave approximation [by dropping all time-dependent terms in Eq. (17)]. Figure 2 displays the steady-state entanglement E_N of the cavity mode d and the mechanical mode b_2 as functions of the coupling asymmetry G_+/G_- for different δ with zero bath occupations for all modes, where the downward triangle denotes the optimal value of each curve. Apparently, E_N is a nonmonotonic function of G_+/G_- in any given set of parameters and takes a maximum for a specific G_+/G_-. The phenomenon is similar to that in Refs. [33,36,40],

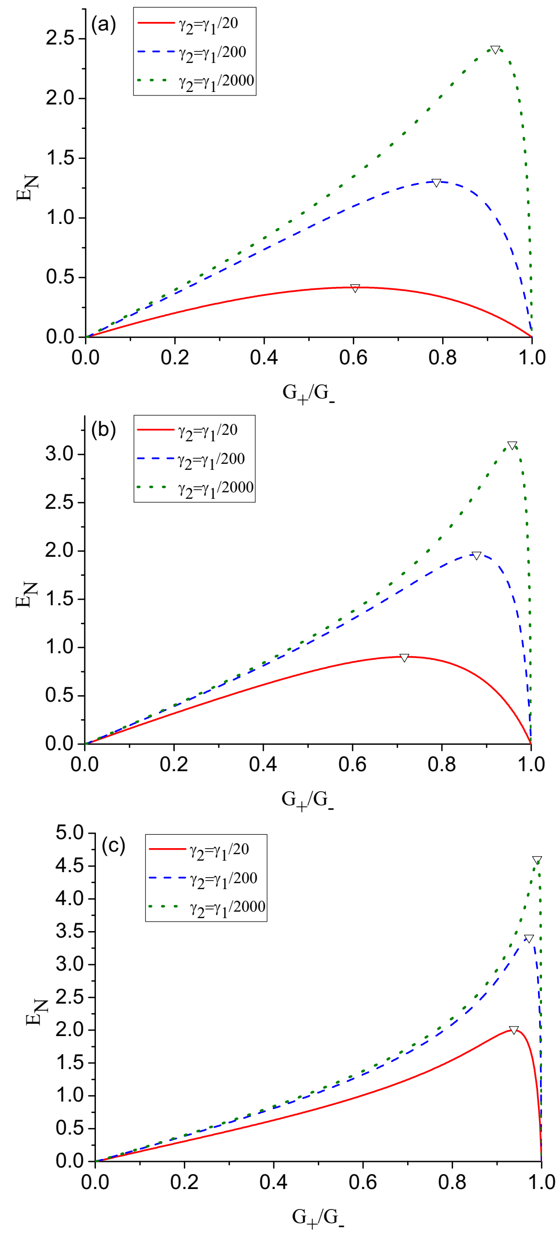

FIG. 2. Stationary cavity-mechanical entanglement E_N vs the ratio of the effective couplings G_+/G_- for different values of δ. (a) $\delta = 10\gamma_1$, (b) $\delta = 5\gamma_1$, and (c) $\delta = \gamma_1$. The other parameters are $G_- = 2.5\gamma_1$, $\kappa = \gamma_2$, and $\bar{n}_1 = \bar{n}_2 = \bar{n}_d = 0$.

and it can be explained as follows. The relation $\tanh r = G_+/G_-$ indicates that the increase of the ratio G_+/G_- can raise the squeezing parameter r, which is beneficial for enhancing the entanglement. But, from another point of view, the increase in G_+/G_- (with G_- fixed) accompanies the decline of effective coupling $G = \sqrt{G_-^2 - G_+^2}$ between the sum mode β_{sum} and the mechanical mode b_1, which is harmful for the cooling effect and thus reduces the amount of entanglement. The best value is obtained when the two competing effects balance. In addition, we find that the smaller the ratio γ_2/γ_1, the lager the maximal entanglement E_N and the optimal G_+/G_- in each figure. Since the entanglement generation is largely

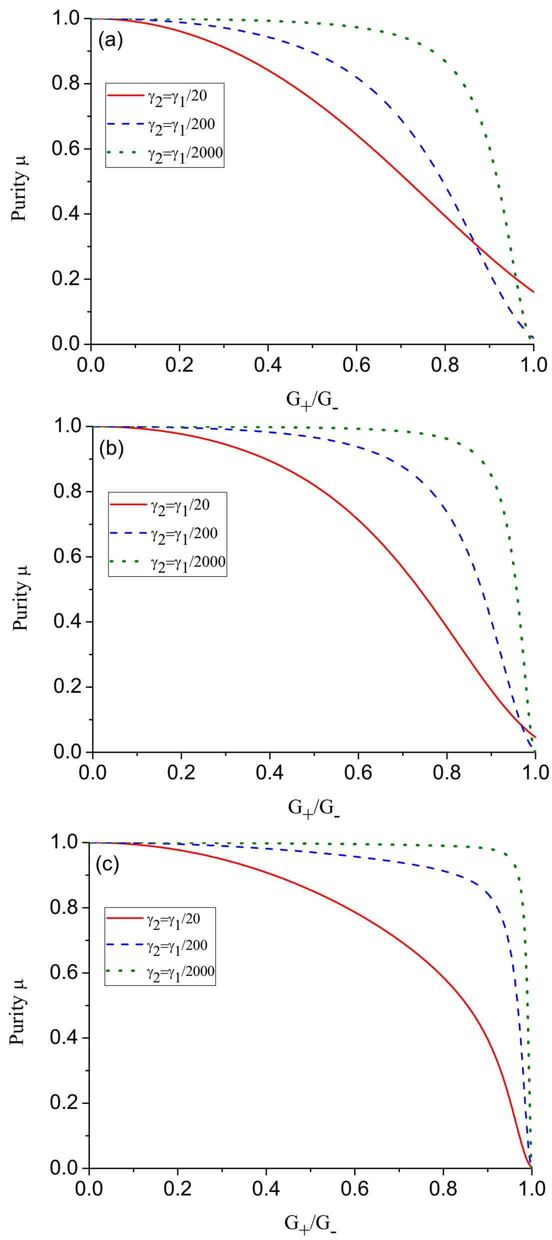

FIG. 3. Steady-state purity of the cavity mode d and the mechanical mode b_2 against the ratio of the effective couplings G_+/G_- for different values of δ. (a) $\delta = 10\gamma_1$, (b) $\delta = 5\gamma_1$, and (c) $\delta = \gamma_1$. All other parameters are the same as those in Fig. 2.

based on cooling the Bogoliubov modes via the dissipative dynamics of the mechanical mode b_1, one would expect that a strong damping rate γ_1 of b_1 and simultaneously weak damping rates γ_2 of b_2 and κ of d should increase the peak entanglement E_N (corresponding to bigger G_+/G_-). Comparing Figs. 2(a), 2(b) and 2(c) with different values of δ, one can find that the achievable entanglement is also dependent on δ, which is the effective coupling between the sum mode β_{sum} and the difference mode β_{diff} and induces the cooling process of β_{diff}.

Figure 3 shows the purity as functions of the coupling asymmetry G_+/G_-. Clearly, we can observe that the purity is inversely correlated to G_+/G_-. If γ_2 is small enough compared

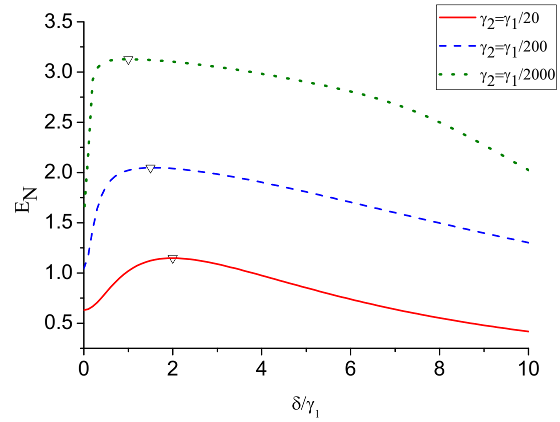

FIG. 4. Stationary cavity-mechanical entanglement E_N vs the effective coupling δ. The sets of parameters corresponding to different lines are $\gamma_2 = \gamma_1/20, G_+/G_- = 0.604$ (red solid line); $\gamma_2 = \gamma_1/200, G_+/G_- = 0.786$ (blue dashed line); and $\gamma_2 = \gamma_1/2000, G_+/G_- = 0.918$ (olive dotted line). The other parameters are $G_- = 2.5\gamma_1$, $\kappa = \gamma_2$, and $\bar{n}_1 = \bar{n}_2 = \bar{n}_d = 0$.

to γ_1, one can keep the high purity (≈ 1) of the steady states over a wide range of G_+/G_-. However, in order to enhance the entanglement, one needs a larger squeezing parameter $r = \tanh^{-1}(G_+/G_-)$ (i.e., larger G_+/G_-) which, on the other hand, weakens the effective coupling $G = \sqrt{G_-^2 - G_+^2}$ and hence cripples the cooling process of Bogoliubov modes toward a pure ground state via the dissipation of b_1. For the sake of gaining a large amount of entanglement while retaining the relatively high purity of the entangled states, we can select proper detuning δ as shown in Figs. 4 and 5, where the downward triangles indicate the optimal values of the corresponding curves. Note that the chosen coupling asymmetry G_+/G_- for each γ_2 is the value where E_N takes the maximum in Fig. 2(a). Remarkably, one can find specific δ where both the entanglement and the purity take the local maximum. For example, when $\gamma_2 = \gamma_1/2000$, $G_+/G_- = 0.918$, and $\delta \approx \gamma_1$, we have $E_N \approx 3.2$ and $\mu \approx 0.98$. In other words, our scheme

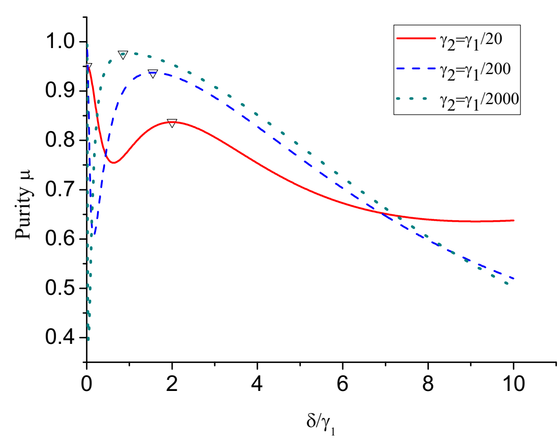

FIG. 5. Steady-state purity of the cavity mode d and the mechanical mode b_2 vs the effective coupling δ. All parameters are the same as those in Fig. 4.

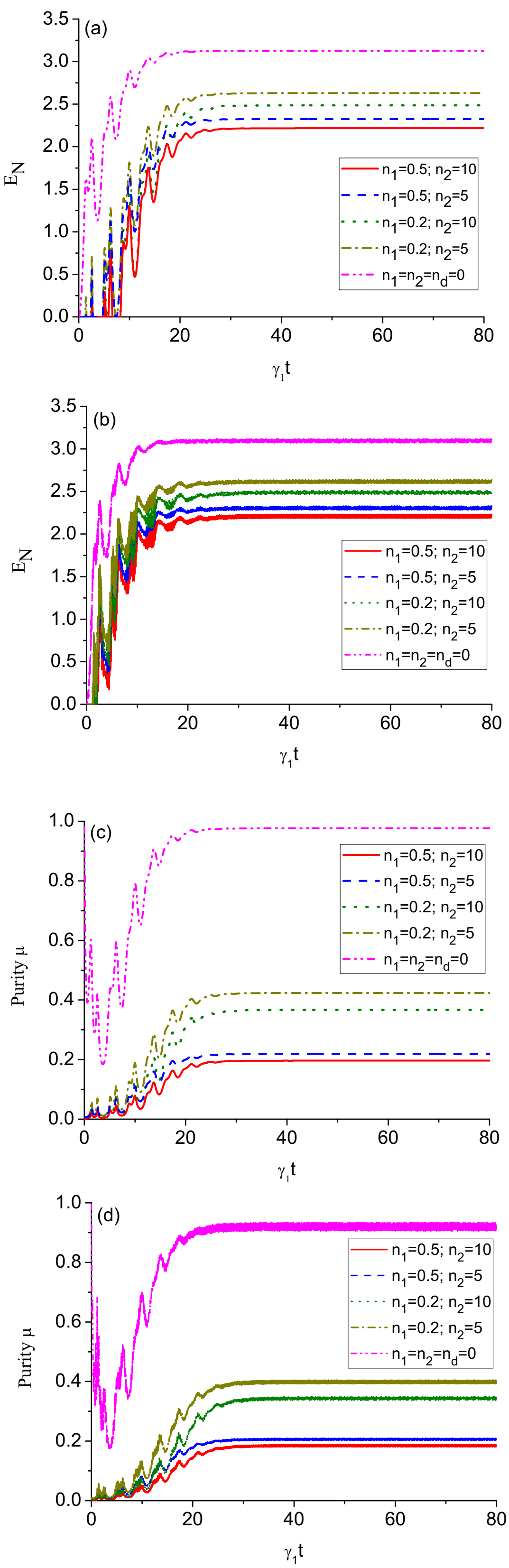

FIG. 6. The time evolution of the entanglement [(a) and (b)] and purity [(c) and (d)] of the quantum states of the cavity mode d and mechanical mode b_2 with [(b) and (d)] and without [(a) and (c)] the nonresonant terms. The parameters are $G_- = 2.5\gamma_1$, $G_+ = 0.918G_-$, $\kappa = \gamma_2 = \gamma_1/2000$, $\delta = \gamma_1$, $\bar{n}_2 = \bar{n}_d$, $\omega_1 = 10\gamma_1$, and $\omega_2 = 100\gamma_1$.

allows the generation of highly pure and strongly entangled optomechanical states.

To find the optimal $\delta_{\rm opt}$, one can recall the Hamiltonian under the rotating-wave approximation in Eq. (11). The sum mode $\beta_{\rm sum}$ is simultaneously coupled to the difference mode $\beta_{\rm diff}$ and the mechanical mode b_1 with beam-splitter-like coupling strengths δ and $\sqrt{2}G$, respectively. The coupling between $\beta_{\rm sum}$ and b_1 induces the cooling process of $\beta_{\rm sum}$, while the coupling between $\beta_{\rm sum}$ and $\beta_{\rm diff}$ is responsible for cooling the $\beta_{\rm diff}$ mode. For a given (fixed) set of parameters G_+, G_-, on the one hand, if δ is too small (relative to $G = \sqrt{G_-^2 - G_+^2}$), $\beta_{\rm diff}$ cannot be effectively cooled by $\beta_{\rm sum}$. For example, when δ approaches 0, only the $\beta_{\rm sum}$ mode can be cooled by b_1. On the other hand, if δ is too large, i.e., $\beta_{\rm diff}$ and $\beta_{\rm sum}$ are strongly coupled, the quanta are confined and swap rapidly between them. Hence, $\beta_{\rm sum}$ cannot be effectively cooled by b_1 in this case. For different sets of parameters G_+ and G_-, one would expect some moderate values of δ that correspond to maximum entanglement and purity. In fact, we have found that the optimal $\delta_{\rm opt}$ is approximately equal to G from Figs. 4 and 5, where $\delta_{\rm opt} \approx G \approx 2\gamma_1$ for red solid lines, $\delta_{\rm opt} \approx G \approx 1.5\gamma_1$ for blue dashed lines, and $\delta_{\rm opt} \approx G \approx 0.99\gamma_1$ for olive dotted lines.

So far all of our discussions have been restricted to the rotating-wave approximation. To study the effects of nonresonant terms of the linearized Hamiltonian in Eq. (5), we plot in Fig. 6 the time evolution of the entanglement and purity with [Figs. 6(b) and 6(d)] and without [Figs. 6(a) and 6(c)] the nonresonant terms for some bath occupancies. We study the system dynamics by numerically solving the differential equation of the covariance matrix in Eq. (20) with the initial states of all modes assumed to be in thermal equilibrium with their local baths. When performing the numerical simulations, the effects of nonresonant terms are included by using the full time-dependent coefficient matrix $M(t)$ in Eq. (17) containing all time-dependent terms. We find that the nonresonant terms only induce small oscillations and do not significantly reduce the amount of steady-state entanglement and purity in the long-time limit, suggesting that the rotating-wave approximation is indeed valid.

IV. CONCLUSIONS

In summary, we have proposed an effective approach to generate pure and strong steady-state optomechanical entanglement (or optical-microwave entanglement) in a hybrid modulated three-mode optomechanical system. By applying a proper two-tone driving of the cavity and modulating coupling strength between two mechanical oscillators (or between a mechanical oscillator and a superconducting transmission line resonator), one can prepare the two target modes of the system in an entangled steady state. The proposal uses an intermediate mechanical mode acting as an engineered reservoir to effectively cool both Bogoliubov modes of the target modes to near their ground state via the beam-splitter-like interactions. Our approach allows the generation of a highly pure and strongly entangled steady state by properly choosing not only the ratio of the effective optomechanical couplings but also the cavity-pump detuning.

ACKNOWLEDGMENTS

C.G.L., H.X., and X.M.L. are supported by the National Natural Science Foundation of China (Grants No. 61275215 and No. 11674059), the Natural Science Foundation of Fujian Province of China (Grants No. 2016J01009 and No. 2013J01008), the Educational Committee of Fujian Province of China (Grants No. JAT160687 and No. JA14397), the 2016 Annual College Funds for Distinguished Young Scientists of Fujian Province of China, and funds from Fujian Polytechnic of Information Technology (Grant No. Y17104). R.X.C. is supported by the Office of Naval Research (Award No. N00014-16-1-3054) and Robert A. Welch Foundation (Grant No. A-1261).

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